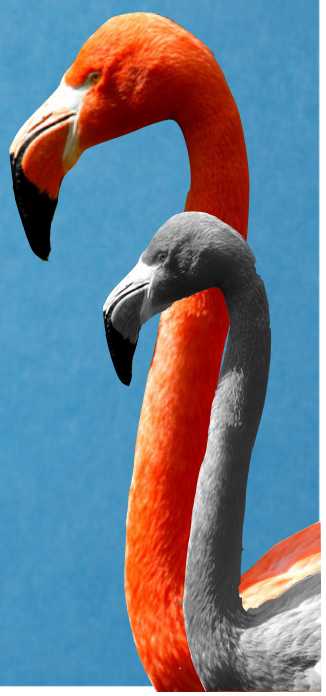
CHAPTER 2
Modeling Distributions of Data

2.2b
Density Curves and Normal Distributions



The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore

Bedford Freeman Worth Publishers

Density Curves and Normal Distributions

Learning Objectives

After this section, you should be able to:

- ✓ ESTIMATE the relative locations of the median and mean on a density curve.
- ✓ ESTIMATE areas (proportions of values) in a Normal distribution.
- ✓ FIND the proportion of *z*-values in a specified interval, or a *z*-score from a percentile in the standard Normal distribution.
- ✓ FIND the proportion of values in a specified interval, or the value that corresponds to a given percentile in any Normal distribution.
- ✓ DETERMINE whether a distribution of data is approximately Normal from graphical and numerical evidence.

Normal Distribution Calculations

We can answer a question about areas in *any* Normal distribution by standardizing and using Table A or by using technology.

How To Find Areas In Any Normal Distribution

Step 1: State the distribution and the values of interest. Draw a Normal curve with the area of interest shaded and the mean, standard deviation, and boundary value(s) clearly identified.

Step 2: Perform calculations—show your work! Do one of the following: (i) Compute a *z*-score for each boundary value and use Table A or technology to find the desired area under the standard Normal curve; or (ii) use the `normalcdf` command and label each of the inputs.

Step 3: Answer the question.

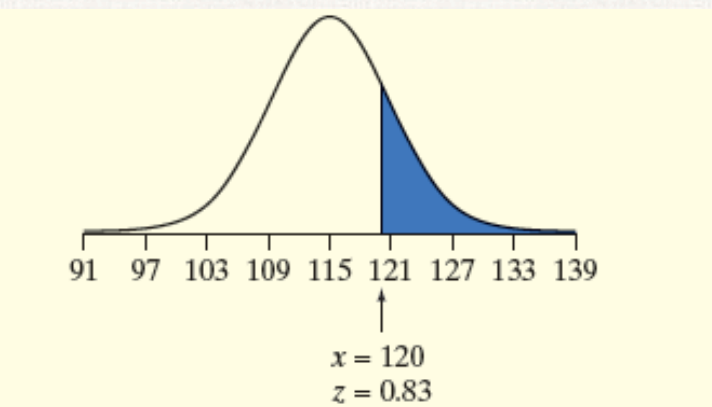
Example: Serving Speed

In a recent tournament, tennis player Rafael Nadal averaged 115 miles per hour (mph) on his serves. Assume that the distribution of his serve speeds is Normal with a standard deviation of 6 mph.

Problem: About what percent of Nadal's serves would you expect to exceed 120 mph?

Step 1: State the distribution and the values of interest. Nadal's serve speed follows a Normal distribution with mean 115 and sd 6 $N(115,6)$.

We want to find the percent of his serves that exceed 120 mph.



Step 2: Perform calculations--show your work.
The standardized score for the boundary value is

$$z = \frac{120 - 115}{6} = .83$$

From Table A, the proportion of *z*-scores above 0.83 is 1 – 0.7967 = 0.2033.

Step 3: Answer the question.
About 20% of Nadal's serves will exceed 120 mph.

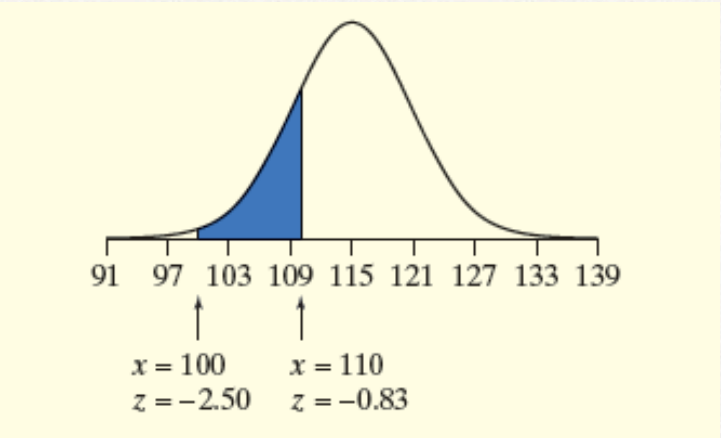
Example: Serving Speed

In a recent tournament, tennis player Rafael Nadal averaged 115 miles per hour (mph) on his serves. Assume that the distribution of his serve speeds is Normal with a standard deviation of 6 mph.

Problem: What percent of Rafael Nadal's serves are between 100 and 110 mph?

Step 1: State the distribution and the values of interest. Nadal's serve speed follows a Normal distribution with mean 115 and sd 6 $N(115,6)$.

We want to find the percent of his serves that are between 100 and 110 mph.



Step 2: Perform calculations--show your work.
The standardized score for the boundary values are

$$z = \frac{110 - 115}{6} = -.83 \; and \; z = \frac{100 - 115}{6} = -2.5$$

From Table A, the proportion of *z*-scores below *z* = –2.50 is 0.0062 and the proportion of *z*-scores below –0.83 is 0.2033. Thus, the proportion of *z*-scores between is 0.2033 – 0.0062 = 0.1971.

Step 3: Answer the question.
About 20% of Nadal's serves are between 100 and 110 mph.

Working Backwards: Normal Distribution Calculations

Sometimes, we may want to find the observed value that corresponds to a given percentile. There are again three steps.

How To Find Values From Areas In Any Normal Distribution

Step 1: State the distribution and the values of interest. Draw a Normal curve with the area of interest shaded and the mean, standard deviation, and unknown boundary value clearly identified.

Step 2: Perform calculations—show your work! Do one of the following: (i) Use Table A or technology to find the value of *z* with the indicated area under the standard Normal curve, then "unstandardize" to transform back to the original distribution; or (ii) Use the invNorm command and label each of the inputs.

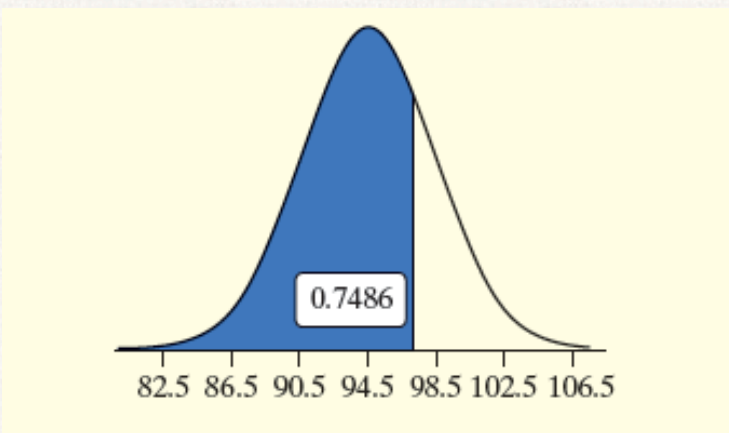
Step 3: Answer the question.

Example: *Heights of three-year-old females*

According to http://www.cdc.gov/growthcharts/, the heights of three-year-old females are approximately Normally distributed with a mean of 94.5 cm and a standard deviation of 4 cm.

Problem: What is the third quartile of this distribution?

Step 1: State the distribution and the values of interest. For three-year-old females, height follows a Normal distribution with mean 94.5 cm and standard deviation 4 cm. Q_3 is the 75th percentile, so we want the boundary value *x* with 75% of the distribution to its left.



Step 2: Perform calculations--show your work. Look in the body of Table A for the value closest to 0.75. A *z*-score of 0.67 gives the closest value (0.7486). Solve $0.67 = \frac{x-94.5}{4}$.

x = 97.18 cm

Step 3: Answer the question. The third quartile of the distribution of three-year-old female heights is about 97.2 cm.

Density Curves and Normal Distributions

Section Summary

In this section, we learned how to…

- ✓ ESTIMATE the relative locations of the median and mean on a density curve.
- ✓ ESTIMATE areas (proportions of values) in a Normal distribution.
- ✓ FIND the proportion of z-values in a specified interval, or a z-score from a percentile in the standard Normal distribution.
- ✓ FIND the proportion of values in a specified interval, or the value that corresponds to a given percentile in any Normal distribution.
- ✓ DETERMINE whether a distribution of data is approximately Normal from graphical and numerical evidence.

Read p. 118-120 ccc 53, 55, 57, 59