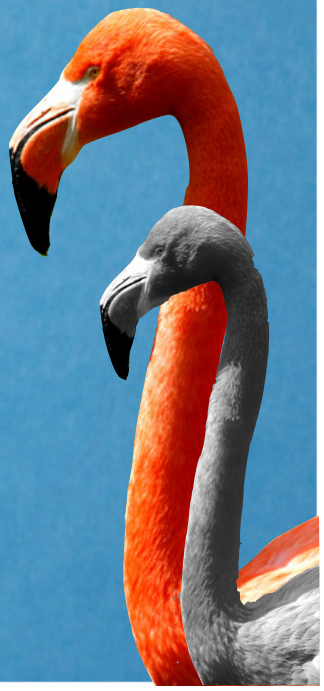
CHAPTER 5
Probability: What Are the Chances?

5.2b
Probability Rules



The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore

Bedford Freeman Worth Publishers

Probability Rules

Learning Objectives

After this section, you should be able to:

- ✓ DESCRIBE a probability model for a chance process.
- ✓ USE basic probability rules, including the complement rule and the addition rule for mutually exclusive events.
- ✓ USE a two-way table or Venn diagram to MODEL a chance process and CALCULATE probabilities involving two events.
- ✓ USE the general addition rule to CALCULATE probabilities.

Basic Rules of Probability

We can summarize the basic probability rules more concisely in symbolic form.

Basic Probability Rules

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

$$P(S) = 1.$$

- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B).$$

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example on page 309. Suppose we choose a student at random. Find the probability that the student (a) has pierced ears.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

(b) is a male with pierced ears.

(c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(a) We want to find *P*(male with pierced ears). There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

(b) We want to find *P*(male or has pierced ears). There are 90 males in the class and 103 students with pierced ears. There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

P(*A* or *B*) = (19 + 71 + 84)/178. So, *P*(*A* or B) = 174/178

Who Owns a Home?

What is the relationship between educational achievement and home ownership? A random sample of 500 U.S. adults was selected. Each member of the sample was identified as a high school graduate (or not) and as a home owner (or not). The two-way table displays the data.

Suppose we choose a member of the sample at random. Find the probability that the member

(a) is a high school graduate

(b) is a high school graduate and owns a home.

(c) is a high school graduate or owns a home.

	High school graduate	Not a high school graduate	**Total**
Homeowner	221	119	**340**
Not a homeowner	89	71	**160**
Total	**310**	**190**	**500**

Define event G as being a high school graduate and event H as being a homeowner.

$$P(G) = \frac{310}{500}$$

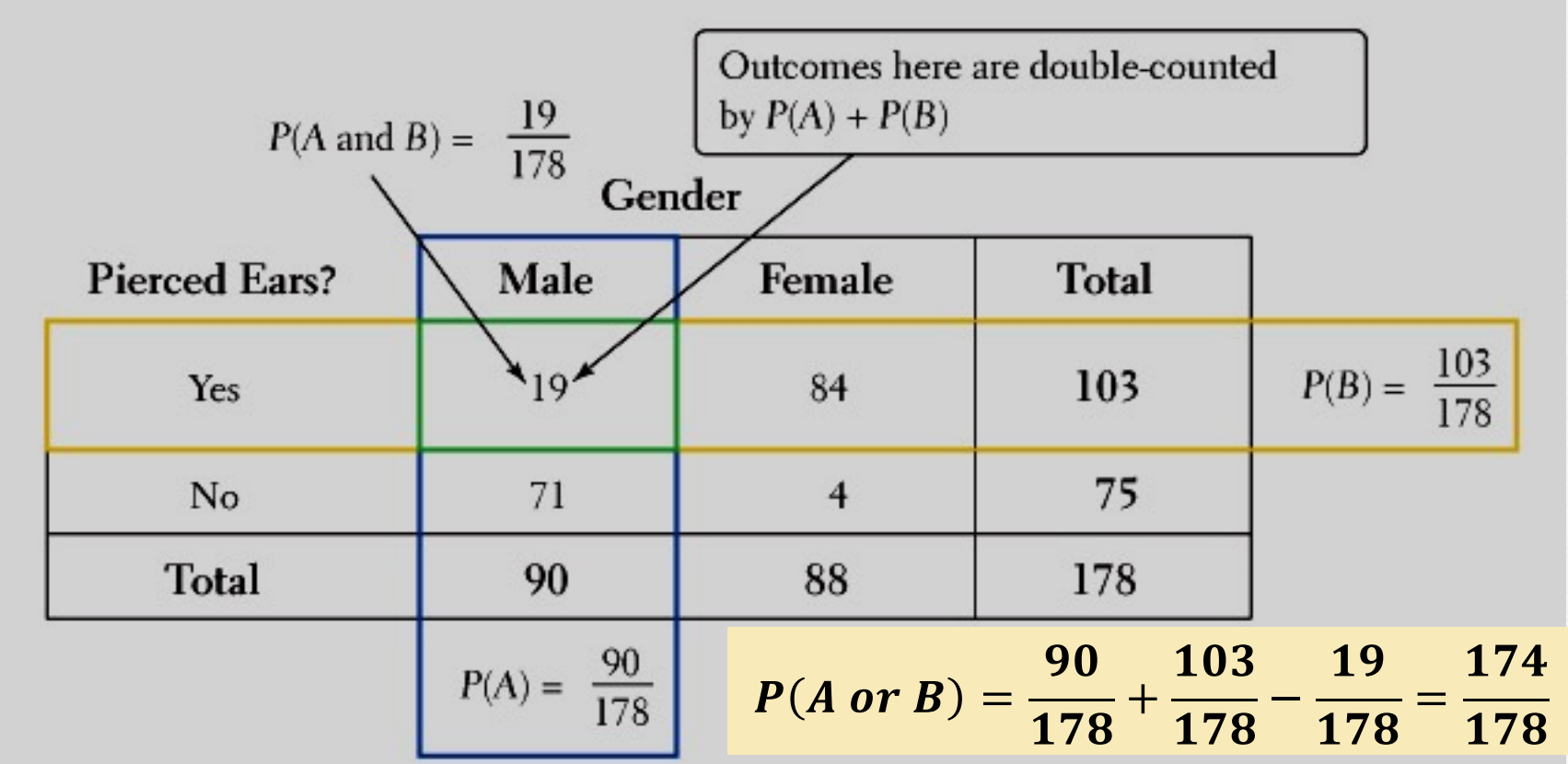
$$P(G \text{ and } H) = \frac{221}{500}$$

$$P(G \text{ or } H) = \frac{221 + 119 + 89}{500} = \frac{429}{500}$$

NOTE: P(G or H) ≠ P(G) + P(H) because G and H are not mutually exclusive.

General Addition Rule for Two Events

We can't use the addition rule for mutually exclusive events unless the events have no outcomes in common.



$$P(A \text{ or } B) = \frac{90}{178} + \frac{103}{178} - \frac{19}{178} = \frac{174}{178}$$

General Addition Rule for Two Events

If *A* and *B* are any two events resulting from some chance process, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Who owns a home?

General Addition Rule for Two Events

If *A* and *B* are any two events resulting from some chance process, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Use the general additional rule to find P(G or H).

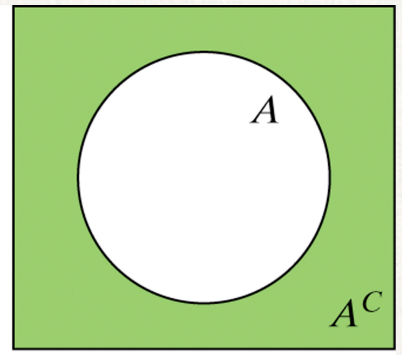
	High school graduate	Not a high school graduate	**Total**
Homeowner	221	119	**340**
Not a homeowner	89	71	**160**
Total	**310**	**190**	**500**

$$P(G \text{ or } H) = \frac{310}{500} + \frac{340}{500} - \frac{221}{500} = \frac{429}{500}$$

Venn Diagrams and Probability

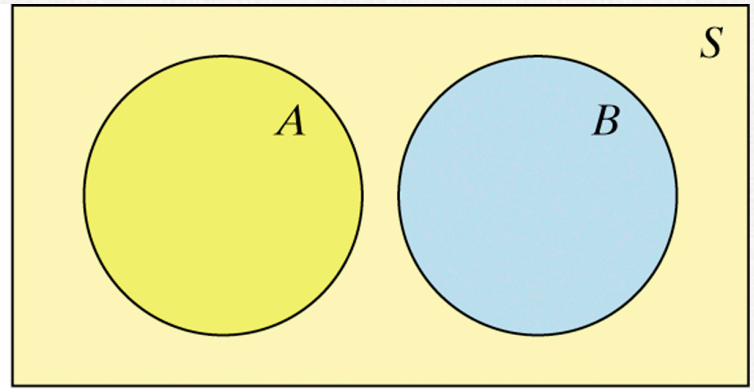
Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in *A*.



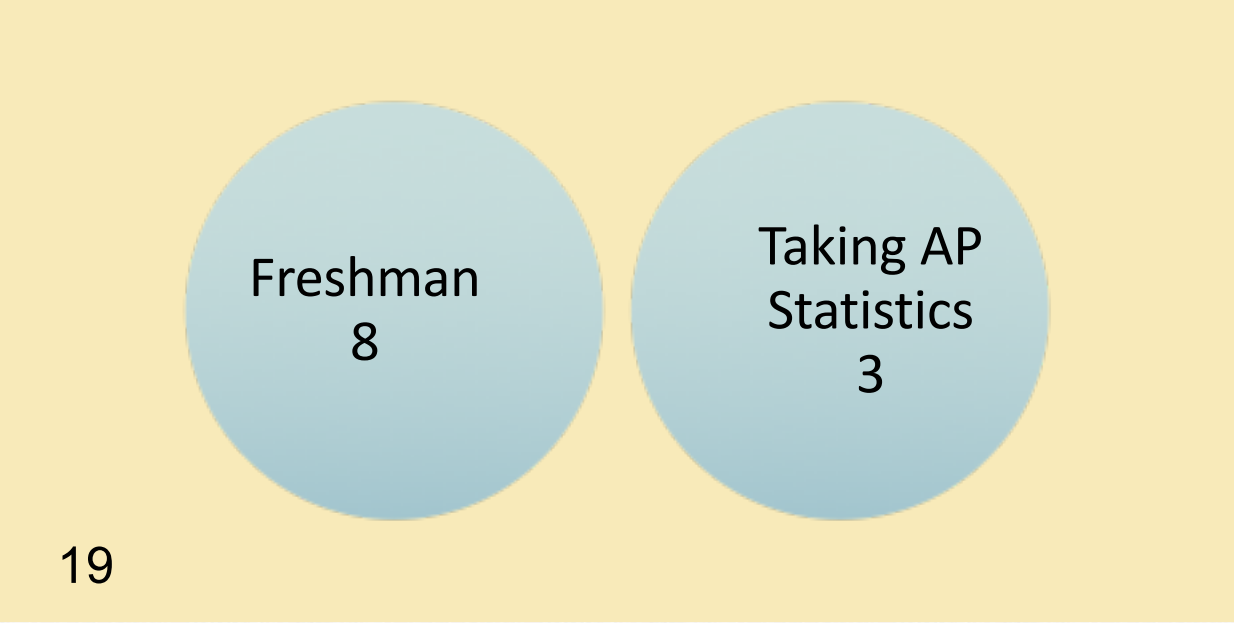
Sample space S is represented by the rectangular box.

The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.



Mutually Exclusive Venn Diagram and Two Way Table

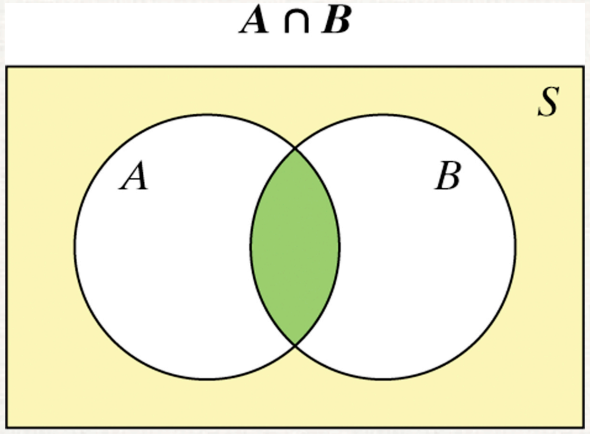
Example: survey of 30 high school students.



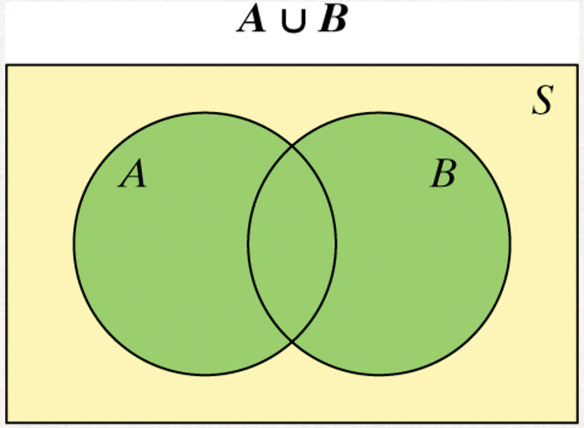
	Freshman	Not a freshman	Total
In AP Statistics	0	3	**3**
Not in AP Statistics	8	19	**27**
Total	**8**	**22**	**30**

Venn Diagrams and Probability

The intersection of events *A* and *B* (*A* ∩ *B*) is the set of all outcomes in both events *A and B.*



The union of events *A* and *B* (*A* ∪ *B*) is the set of all outcomes in either event *A or B.*

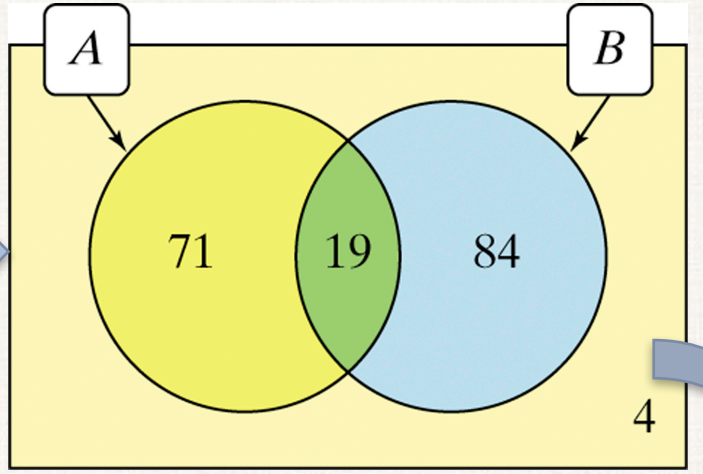


Hint: To keep the symbols straight, remember ∪ for **u**nion and ∩ for i**n**tersection.

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

Who Owns a Home? Venn diagram

Make a Venn diagram to represent the two-way table summarizing the relationship between education status and home ownership. Our events of interest were G: is a high school graduate and H: is a homeowner.

	High school graduate	Not a high school graduate	**Total**
Homeowner	221	119	**340**
Not a homeowner	89	71	**160**
Total	**310**	**190**	**500**

Phone usage

According to the National Center for Health Statistics (http://www.cdc.gov/nchs/data/nhis/earlyrelease/wireless201306.pdf), in December 2012, 60% of U.S. households had a traditional landline telephone, 89% of households had cell phones, and 51% had both. Suppose we randomly selected a household in December 2012.

(a) Make a two-way table that displays the sample space of this chance process.

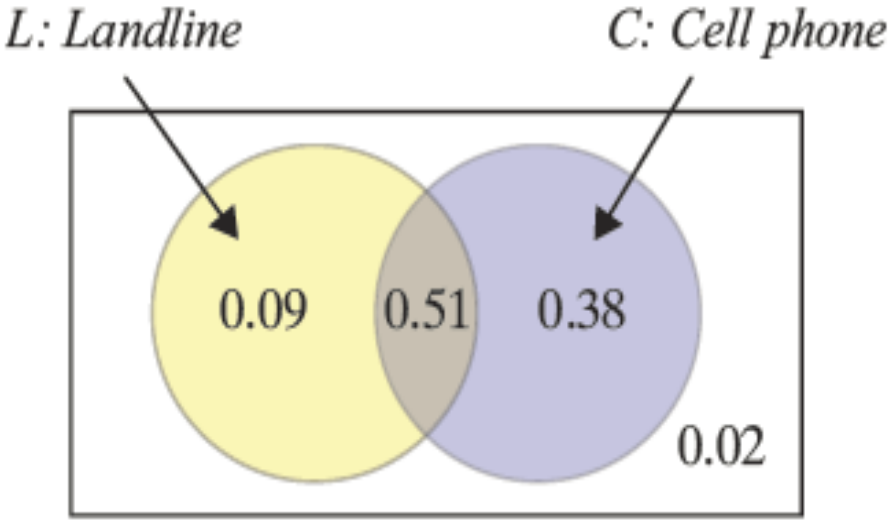
	Cell phone	No cell phone	Total
Landline	0.51	0.09	**0.60**
No landline	0.38	0.02	**0.40**
Total	**0.89**	**0.11**	**1.00**

Phone usage

According to the National Center for Health Statistics, in December 2012, 60% of U.S. households had a traditional landline telephone, 89% of households had cell phones, and 51% had both. Suppose we randomly selected a household in December 2012.

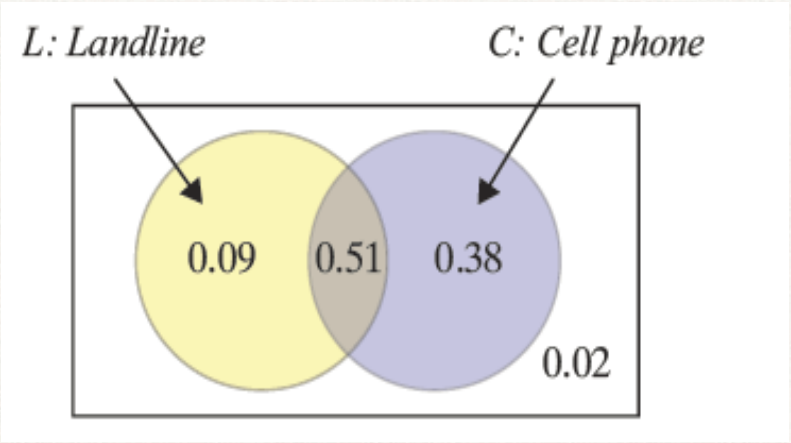
(b) Construct a Venn diagram to represent the outcomes of this chance process.

	Cell phone	No cell phone	**Total**
Landline	0.51	0.09	**0.60**
No landline	0.38	0.02	**0.40**
Total	**0.89**	**0.11**	**1.00**



Phone usage

	Cell phone	No cell phone	**Total**
Landline	0.51	0.09	**0.60**
No landline	0.38	0.02	**0.40**
Total	**0.89**	**0.11**	**1.00**



(c) Find the probability that the household has at least one of the two types of phones.

$$P(L \cup C) = P(L) + P(C) - P(L \cap C)$$
$$= 0.60 + 0.89 - 0.51 = 0.98$$

There is a 0.98 probability that the household has a least one of the two types of phones.

(d) Find the probability that the household has a cell phone only.

P(cell phone only) = $P(L^C \cap C) = 0.38$

Probability Rules

Section Summary

In this section, we learned how to…

- ✓ DESCRIBE a probability model for a chance process.
- ✓ USE basic probability rules, including the complement rule and the addition rule for mutually exclusive events.
- ✓ USE a two-way table or Venn diagram to MODEL a chance process and CALCULATE probabilities involving two events.
- ✓ USE the general addition rule to CALCULATE probabilities.

- ✓ Read p. 309-314 ccc 29, 33-36, 49, 51, 53, 55