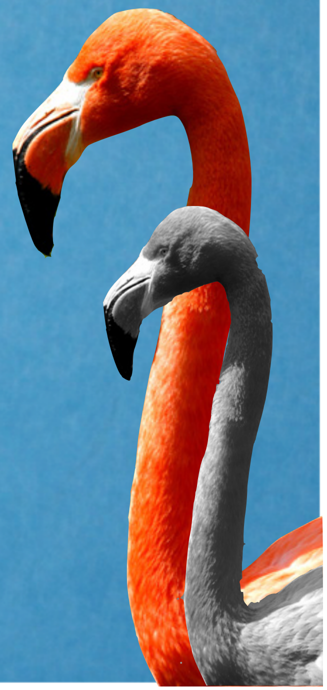
CHAPTER 6
Random Variables

6.3c
Binomial and Geometric Random Variables



The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore

Bedford Freeman Worth Publishers

Binomial and Geometric Random Variables

Learning Objectives

After this section, you should be able to:

- ✓ DETERMINE whether the conditions for using a binomial random variable are met.
- ✓ COMPUTE and INTERPRET probabilities involving binomial distributions.
- ✓ CALCULATE the mean and standard deviation of a binomial random variable. INTERPRET these values in context.
- ✓ FIND probabilities involving geometric random variables.

Is This Your Lucky Day?

Let's say you came to class and I gave you a choice. Option 1: You get 10 homework problems assigned. Option 2: Play the Lucky Day Game.

How does it work? A student will be selected at random and asked to pick a number 1 through 7. The teacher will use technology to randomly choose the lucky number. If the student picks the correct number, the class will only have one homework problem. If the student picks the wrong number, the process starts again. A randomly chosen student chooses a number 1 through 7, the teacher will use technology to randomly choose a number (so it could be the same as the first time). If this student is wrong, you will have two homework problems. The game continues until someone correctly guesses the lucky number. Your teacher will assign the number of homework problems that is equal to the total number of guesses made by members of your class.

Would you play?   You have 30 seconds to decide!

Binomial Settings

When the same chance process is repeated several times, we are often interested in whether a particular outcome does or doesn't happen on each repetition. Some random variables count the number of times the outcome of interest occurs in a fixed number of repetitions. They are called **binomial random variables.**

A **binomial setting** arises when we perform several independent trials of the same chance process and record the number of times that a particular outcome occurs. The four conditions for a binomial setting are:

B • **B**inary? The possible outcomes of each trial can be classified as "success" or "failure."

I • **I**ndependent? Trials must be independent; that is, knowing the result of one trial must not tell us anything about the result of any other trial.

N • **N**umber? The number of trials n of the chance process must be fixed in advance.

S • **S**uccess? There is the same probability p of success on each trial.

Geometric Settings

In a binomial setting, the number of trials n is fixed and the binomial random variable X counts the number of successes.

In other situations, the goal is to repeat a chance behavior until a success occurs. These situations are called **geometric settings**.

A **geometric setting** arises when we perform independent trials of the same chance process and record the number of trials it takes to get one success. On each trial, the probability p of success must be the same.

Geometric Settings

In a geometric setting, if we define the random variable Y to be the number of trials needed to get the first success, then Y is called a **geometric random variable.** The probability distribution of Y is called a **geometric distribution.**

The number of trials Y that it takes to get a success in a geometric setting is a **geometric random variable**. The probability distribution of Y is a geometric distribution with parameter p, the probability of a success on any trial. The possible values of Y are 1, 2, 3, ...

Like binomial random variables, it is important to be able to distinguish situations in which the geometric distribution does and doesn't apply!

Geometric Probability Formula

The Lucky Day Game. The random variable of interest in this game is Y = the number of guesses it takes to correctly match the lucky number. What is the probability the first student guesses correctly? The second? Third? What is the probability the k^{th} student guesses correctly?

$$P(Y = 1) = 1/7$$

$$P(Y = 2) = (6/7)(1/7) = 0.1224$$

$$P(Y = 3) = (6/7)(6/7)(1/7) = 0.1050$$

Geometric Probability Formula

If Y has the geometric distribution with probability p of success on each trial, the possible values of Y are 1, 2, 3, … . If k is any one of these values,

$$P(Y = k) = (1 - p)^{k-1} p$$

Monopoly

In the board game Monopoly, one way to get out of jail is to roll doubles. Suppose that this was the only way a player could get out of jail. The random variable of interest in this example is Y = number of attempts it takes to roll doubles one time.

Verify that this is a geometric random variable.

Independent trials of the same chance process: Each attempt is one trial of the chance process. Knowing the outcome of previous rolls does not tell us anything about future rolls.

On each trial the probability of success must be the same: On each roll, the probability of success is 1/6.

Record the number of trials it takes to get a success: Because Y counts the number of attempts it takes to get doubles, it is a geometric random variable with parameter p = 1/6.

This is a geometric setting.

Monopoly

Let the random variable *Y* be defined as in the previous slide.

(a) Find the probability that it takes 3 turns to roll doubles.

$$P(Y = 3) = \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) = \mathbf{0.116}$$

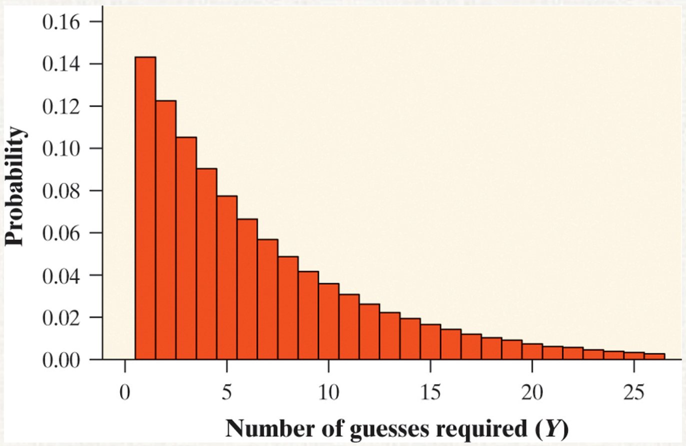
(b) Find the probability that it takes more than 3 turns to roll doubles, and interpret this value in context.

$$\begin{aligned} P(Y > 3) &= 1 - P(Y \leq 3) \\ &= 1 - P(Y = 3) - P(Y = 2) - P(Y = 1) \\ &= 1 - \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^1 \left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^0 \left(\frac{1}{6}\right) \\ &= \mathbf{0.5787} \end{aligned}$$

If a player tried to out of out jail many, many times by trying to roll doubles, about 58% of the time it would take more than 3 attempts.

Mean of a Geometric Random Variable

The table below shows part of the probability distribution of *Y*. We can't show the entire distribution because the number of trials it takes to get the first success could be an incredibly large number.



y_i	1	2	3	4	5	6	…
p_i	0.143	0.122	0.105	0.090	0.077	0.066	

Shape: The heavily right-skewed shape is characteristic of any geometric distribution. That's because the most likely value is 1.

Center: The mean of *Y* is $\mu_Y = 7$. We'd expect it to take 7 guesses to get our first success.

Spread: The standard deviation of *Y* is $\sigma_Y = 6.48$. If the class played the Lucky Day game many times, the number of homework problems the students receive would differ from 7 by an average of 6.48.

Mean (Expected Value) Of A Geometric Random Variable

If *Y* is a geometric random variable with probability p of success on each trial, then its mean (expected value) is $E(Y) = \mu_Y = 1/p$.

Binomial and Geometric Random Variables

Section Summary

In this section, we learned how to…

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- ✓ CALCULATE the mean and standard deviation of a binomial random variable. INTERPRET these values in context.
- ✓ FIND probabilities involving geometric random variables.

- ✓ Read p. 404-410 ccc 93, 95, 97, 99, 101-104